# On the Galerkin Technique for Evaluating the Conductor Loss in Microstrip Reflectarrays

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***Abstract*— Integral equations for the analysis of microstrip reflectarrays consisting of thin perfectly conducting patches generally employ edge conditions in the basis functions for good convergence. The finite conductivity of a practical structure is treated as a perturbation by using the well-known Leontovich boundary condition. The Galerkin technique for the latter results in diverging integrals in moment matrix elements corresponding to edge conditions in basis functions approaching infinity across the current flow direction. Previously a criterion to stop the evaluation of the diverging integrals at a distance from the edge was proposed. In this paper we show that excellent results may be achieved by simply eliminating relevant edge conditions in the testing functions in the moment method.**

*Index Terms*—Method of Moments (MoM), Galerkin Technique, Reflectarray, Edge Conditions, Microstrip Patch

## I. INTRODUCTION

The infinite array model for reflectarray elements, based on local periodicity is popular in the analysis and design of reflectarrays [1]. In this model, integral equations are formulated for induced currents in the unit cell of a doubly periodic infinite array excited by a plane wave. In order to make the problem tractable, we assume that the reflectarray patches and the ground plane are perfect conductors of zero thickness. The integral equations are solved by the method of moments [1-3]. It was shown that the method of moments analysis using a single basis function could provide good solution for a reflectarray of rectangular patches, whereas for accurate results over a wide range of values of physical parameters, many trigonometric basis functions with edge conditions are needed [2, 3]. The edge conditions for thin perfectly conducting rectangular patches are such that the current perpendicular to the edge approaches zero while that parallel to the edge approaches infinity.

The basis functions for the x-directed current, $J_x$ are in the form

$$\begin{pmatrix} \cos(m\pi x/a) \\ \sin\{(m+1)\pi x/a\} \end{pmatrix} \begin{pmatrix} \cos(n\pi y/b) \\ \sin\{(n+1)\pi y/b\} \end{pmatrix} \bullet [1-(2x/a)^2]^{-1/2} [1-(2y/b)^2]^{-1/2} \quad (1)$$

where m=1,3,5 etc. and n=0,2,4 etc. Fig. 1 shows the dimensions a and b of a rectangular patch in the unit cell of an infinite array. It is found that two odd and two even variations along each direction yield excellent accuracy with sixteen unknown coefficients for a non-separable distribution for $J_x$ [2, 3]. For the y-directed current, the basis functions are found by replacing x, y, a, and b by y, x, b, and a respectively in (1).

In order to determine the power loss in the conductor, the Leontovich boundary condition is employed to model thin imperfectly conducting patches, and a perturbation method is used to formulate the integral equations [2]. In the perturbation method, the basis functions for the imperfect conductors are the same as those for the perfect conductors. For accurate solution of the integral equations, it is common practice to choose the testing functions the same as the basis functions, i.e., the Galerkin technique [4]. It has been shown that the Galerkin technique for the imperfect conductor yields diverging integrals for certain moment matrix elements [2]. The divergence problem was obviated by stopping such integrals at a distance $\delta$ = w/220 from the edge, where w is the size of the patch across the current direction [2]. The method of moments using the stopping criterion is denoted $MoM_2$ [2].

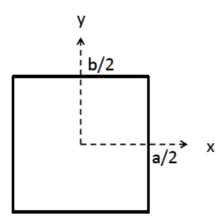


Fig. 1 A rectangular patch of dimension a x b.

In this work, based on a numerical study we determined that the testing functions without edge conditions across the current directions yield accurate results for the characteristics of reflectarrays without producing any diverging integrals. Such testing functions for $J_x$ are in the form

$$\begin{pmatrix} \cos(m\pi x/a) \\ \sin\{(m+1)\pi x/a\} \end{pmatrix} \begin{pmatrix} \cos(n\pi y/b) \\ \sin\{(n+1)\pi y/b\} \end{pmatrix} \bullet [1-(2x/a)^2]^{-1/2} \quad (2)$$

Note that the y-dependent edge condition term in (1) is suppressed in (2). The trigonometric functions in y-dependent part in (2) are non-zero and finite at the edges y=$\pm$b/2. Similarly, the x-dependent part of the testing function $J_y$ will have trigonometric functions that are non-zero and finite at edges x=$\pm$a/2. The method of moments using basis functions (1) and testing functions (2) for $J_x$ , and similar terms for $J_y$ is denoted MoM-T in this paper. Testing functions with finite non-zero values at the edges can provide good accuracy for the edge behavior approaching infinity. This procedure is similar to the conventional perturbation method to find the power loss in an imperfect conductor where we use the results of the current in the perfect conductor with its intrinsic resistance. Numerous results computed in this work for reflectarrays of different geometrical and material parameters using MoM-T showed that the testing functions used in this work provide excellent accuracy. Some representative results are discussed in the next section.

## II. Computed and Measured Results

The results computed for $MoM_2$ for all cases in [2] are repeated here for MoM-T. For completeness Table 1 showing the parameters of reflectarray antennas discussed in [2] is reproduced here. In case A square patches of side 0.2162 cm were used. Figs. 2 and 3 compare the measured and computed values of the reflection coefficient phase and magnitude, respectively. Reflection coefficients were measured for the case of normal incidence from a reflectarray consisting of identical square patch elements [2]. Excellent agreement between measured phase and that computed by MoM-T and $MoM_2$ are found in Fig. 2. The magnitudes of MoM-T and $MoM_2$, are typically within 0.1 dB of the measured values of reflection coefficient magnitude.

The ground plane is assumed to be a perfect conductor and the surface impedance of the patch is doubled in both $MoM_2$ and MoM-T. Thus, the conductor loss in the rectangular patch is equal to that of the ground plane. This procedure, proposed by Mosig in [5], is justified by the cavity model for a patch antenna [6].

Table 1 Reflectarray parameters for two cases (*Reproduced from [2] with permission from the Electromagnetics Academy*)

| Parameters | Case A | Case B |
|---|---|---|
| Nominal resonant frequency | 35.75 GHz | 13.285 GHz |
| Lattice spacing | 0.4191 cm | 1.1291 cm |
| Substrate thickness | 0.0381 cm | 0.08128 cm |
| Substrate dielectric constant | 2.9503 | 3.58 |
| Loss tangent | 0.0012 | 0.0027 |
| Angles of incidence of the plane wave (θ, ϕ) | $(0^o,0^o)$ | $(30^o,0^o)$ |

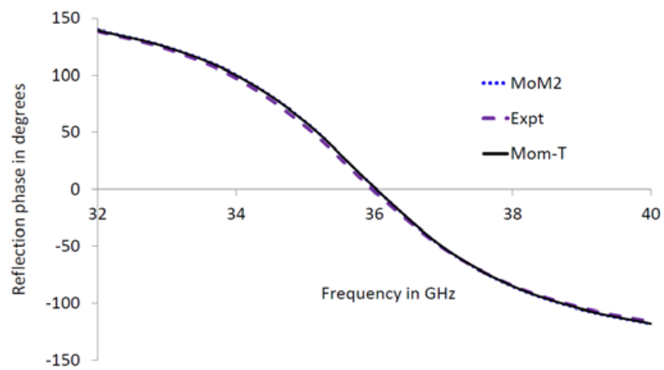


Fig. 2 Computed and measured values of the reflection coefficient phase for normal incidence.

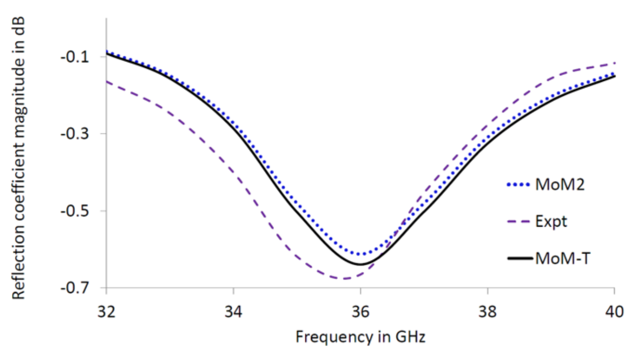


Fig. 3 Computed and measured values of the reflection coefficient magnitude for normal incidence (Case A).

Fig. 4 shows the reflection coefficient magnitude at resonance as a function of substrate thickness for Case A. Very good agreement is found between the values computed by MoM-T and $MoM_2$ for the substrate thickness down to 0.025 wavelength in the substrate material. The resonant frequency given in Table 2 is found to decrease as the substrate thickness increases. Values of the resonant frequency computed by MoM-T and $MoM_2$, shown in Table 2, agree to within 0.025%. Reflection coefficient magnitudes computed using MoM-T for other values of substrate permittivity, not shown here, also showed good agreement with the corresponding results of $MoM_2$.

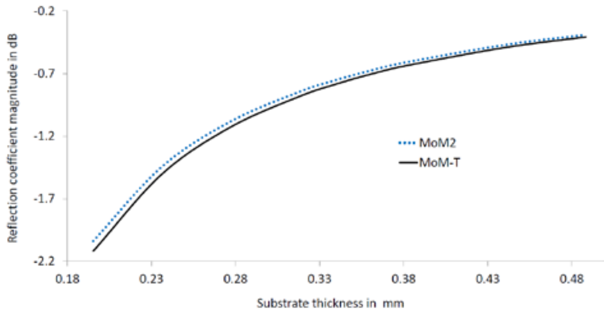


Fig. 4 The reflection coefficient magnitude versus substrate thickness for Ka band.

Table 2. Resonant frequency in GHz as a function of substrate thickness computed by MoM-T and $MoM_2$ (case A in Table 1).

| Substrate thickness in mm | Resonant frequency $MoM_2$ δ=w/220 | Resonant frequency MoM-T |
|---|---|---|
| 0.1954 | 37.830 | 37.930 |
| 0.3175 | 36.592 | 36.610 |
| 0.381 | 36.026 | 36.027 |
| 0.4396 | 35.539 | 35.530 |
| 0.4885 | 35.156 | 35.140 |

Figure 5 shows the reflection coefficient magnitude at resonance versus substrate thickness for a Ku band reflectarray (Case B in Table 1). The side of each unit cell square patch is 0.542 cm. A $TM_z$ plane wave is incident at an angle of $\theta = 30^o$ and $\phi = 0^o$. Figs. 4 and 5 show that the loss increases rapidly as the substrate thickness decreases below approximately 0.066 wavelength in the substrate material since the patches exhibit high quality factor.

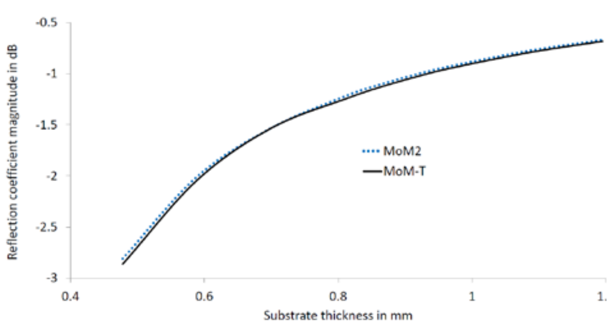


Fig. 5 The reflection coefficient magnitude versus substrate thickness for Ku band.

Table 3 Resonant frequency and total loss as a function of the dielectric loss tangent computed by MoM-T and $MoM_2$ (case A in Table 1 with square patches of side 0.216 cm)

| Dielectric loss tangent | $MoM_2$, δ=w/220 Resonant Frequency (GHZ) | $MoM_2$, δ=w/220 Total loss (dB) | MoM-T Resonant Frequency (GHZ) | MoM-T Total loss (dB) |
|---|---|---|---|---|
| 0.0012 | 36.03 | 0.61 | 36.061 | 0.64 |
| 0.005 | 36.03 | 1.18 | 36.061 | 1.21 |
| 0.01 | 36.03 | 1.94 | 36.06 | 1.96 |
| 0.02 | 36.00 | 3.48 | 36.056 | 3.49 |

Table 3 shows computed results for the resonant frequency and the total loss at resonance for reflectarrays of square patches of side 0.2162 cm. All other parameters are specified in Table 1, case A. The results show that the resonant frequencies are independent of the dielectric loss tangent. The total loss computed by the two methods is in good agreement, thereby demonstrating that MoM-T works for a wide range of values for the dielectric loss tangent as well.

Figs. 6 through 11 show the reflection coefficient magnitude and phase as a function of frequency for the Ka band reflectarray (Case A in Table 1) with an added 0.0127 cm thick Kapton superstrate of dielectric constant 3.0 and loss tangent 0.001. The square patch size is 0.2025 cm. MoM-T is found to be in good agreement with HFSS [7] and so is $MoM_2$. Figs. 6 and 7 show the results for the normal incidence. Figs. 8 and 9 correspond to $\theta = 45^o$ and $\phi = 0^o$ for TE polarization while Figs. 10 and 11 present the results for $\theta = 45^o$ and $\phi = 0^o$ for TM polarization. Very good agreement between MoM-T and HFSS are found for all cases. The discrepancy between MoM-T and HFSS for the reflection coefficient magnitude is typically within a few hundredths of a dB in all cases. Results computed using MoM-T are in better agreement with those of HFSS compared to $MoM_2$, for the reflection phase.

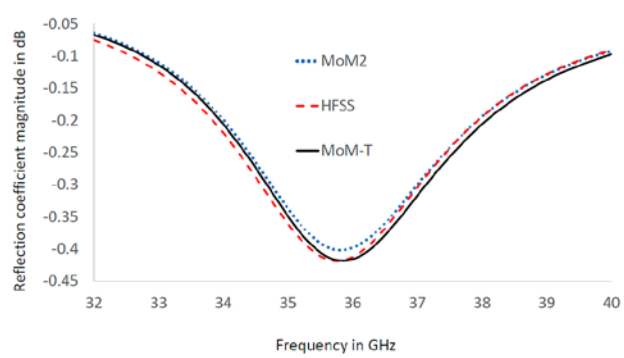


Fig. 6 The reflection coefficient magnitude for a reflectarray with a superstrate at normal incidence.

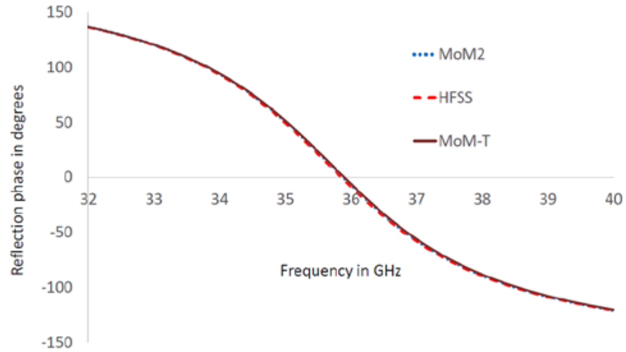


Fig. 7 The reflection coefficient phase for a reflectarray with a superstrate at normal incidence.

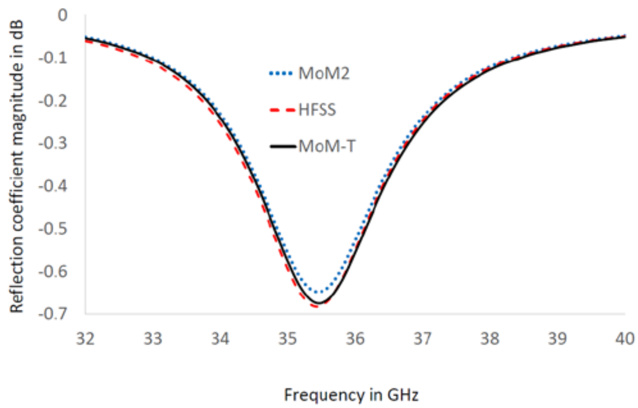


Fig. 8 The reflection coefficient magnitude for a reflectarray with a superstrate for TE polarization at 45$^o$.

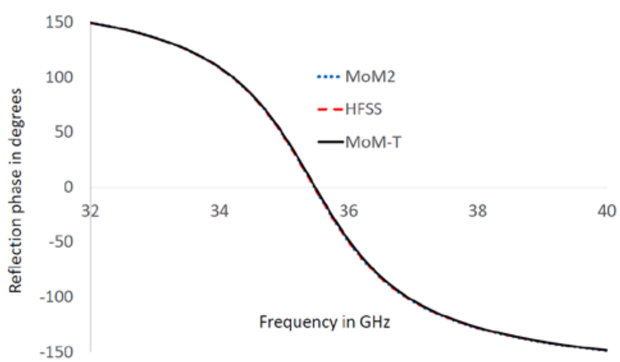


Fig. 9 The reflection coefficient phase for a reflectarray with a superstrate for TE polarization at 45$^o$.

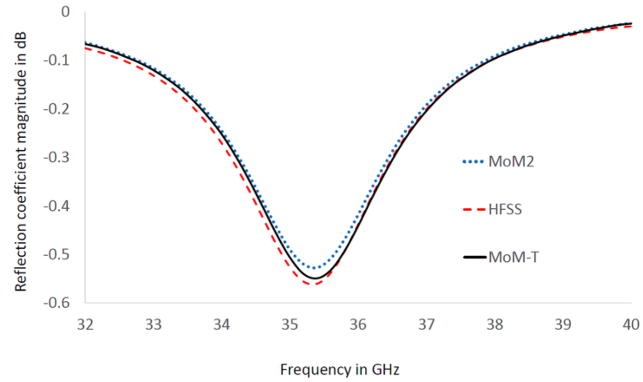


Fig. 10 The reflection coefficient magnitude for a reflectarray with a superstrate for TM polarization at 45$^o$.

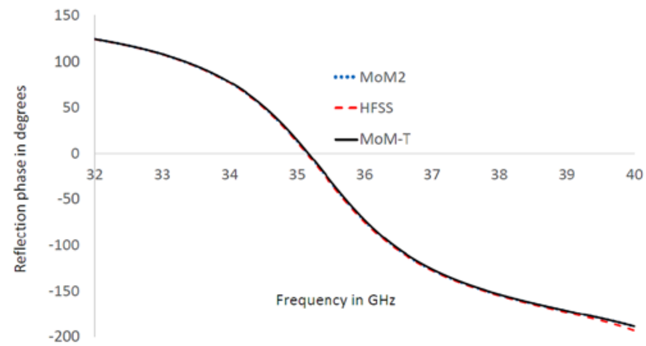


Fig. 11 The reflection coefficient phase for a reflectarray with a superstrate for TM polarization at 45$^o$.

The losses computed by MoM-T typically are within a few hundredths of a dB of the results computed by HFSS for all cases. The resonant frequencies computed by MoM-T for all cases are within about 0.3% of the value computed by HFSS. MoM-T is in excellent agreement with HFSS for resonant frequencies. Thus, the use of testing functions without the edge condition across the current direction is an excellent choice for accurate evaluation of the reflection coefficient of reflectarrays for all cases of substrate thickness, permittivity, dielectric loss tangent, polarization, angles of incident plane waves and for superstrates as well.

## III. Conclusions

The diverging integrals encountered in the Galerkin technique employing basis and testing functions with edge conditions approaching infinity for reflectarrays consisting of thin rectangular patches have been eliminated by using testing functions without the edge conditions across the current direction. Such a technique is found to work well, yielding reflection coefficient magnitude within 0.1 dB of measured or HFSS results for a range of values of dielectric constant, substrate thickness and for superstrates as well. Excellent results are also obtained for the reflection coefficient phase and hence, the resonant frequency. This method eliminates the need for a stopping criterion used in a previous work. The method of doubling the surface impedance of the patch to account for the loss in the ground plane has been found to be valid.

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