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Resonance Effects in Material and Metamaterial Coated Antennas and Resolution of Anomalous Power Radiations

L. Shafai

University of Manitoba, Dept. of Electrical and Computer Engineering,

Winnipeg, Manitoba, Canada, R3T 5V6

(Email: Lot.Shafai@umanitoba.ca)

***Abstract*—Conducting antennas are often coated with insulating materials to minimize the environmental effects, or prevent electrical shorting in conducting media. In most cases the coating material is too thin, with respect to the wavelength, to cause appreciable changes in the antenna performance. However, in electrically large antennas, or at high frequencies, the coating thickness can be sufficient to cause resonances that can influence the antenna performance. Previous studies have shown enhanced power radiations due to such resonances. This article is aimed at reviewing such cases and providing proper explanation of the resonance phenomena and the resolution of anomalous power radiations. To make the problem more realistic, yet analytically solvable to prevent computational errors, spherical dipole antennas are selected and solutions are developed for both single and double coating configurations. In the former case, only material coating is considered, and a parametric study in performed to investigate the resonance effects on the antenna performance and radiated power, as well as, the directivity. However, in the latter case the material and metamaterial pairs can cause additional resonances, and their combination is used for a comprehensive study of the coating effects. It is shown that the cause of the anomalous power radiation is the use of idealized sources that leads to incorrect interpretation of the results, rather than the enhanced power radiation due to the coatings. It is also shown that the phenomenon is the same for both natural materials and metamaterials, providing identical results.**

***Index Terms*—Spherical antennas, Coated antennas, Resonant modes, Metamaterials, Power enhancements, Anomalous power radiation.**

I. INTRODUCTION

The spherical antennas and radiation from slots on conducting or coated conducting spherical objects, have been investigated by a number of authors. Karr [1] has studied the size and position of the slot on the radiation characteristics of the slotted spherical antenna. Later Mushiake and Webster [2] studied the impedance and radiation characteristics of asymmetrically fed slots on a conducting sphere. Harrington [3] has also studied spherical antennas, and provided an analytic solution, in terms of the vector potentials, for the radiated field and the power. Kerker [4] has provided analytic solutions for spherical and multilayer-coated spherical objects. The general problem of coated spherical antennas, excited by an azimuthal slot, was investigated by Towaij and Hamid [5]. They have considered multiple coatings and studied the radiation characteristics of the antenna, but have not specifically investigated the coating effect on the radiation power. This study was conducted later by Shafai and Chugh [6], who investigated the effect of coating on the radiated power of a slotted spherical antenna, coated by a homogeneous material. They have shown that the radiated power, initially, increases gradually by increasing the coating thickness, then, increases rapidly at the mode resonances inside the coating material. This study was undertaken after an initial investigation by Shafai [7], on slotted cylindrical antennas, coated by homogeneous materials. The coating effect on the radiated power was found to be similar in both cases. Independent of these studies on material coatings, resonance effects were also studied in paired material/metamaterial shells over small ideal dipole antennas [8], [9], in subsequent years, indicating enhanced power radiation. These latter investigations attributed the enhanced power radiation to metamaterials. The effects are, however, similar to the case of slot antennas coated with homogeneous materials, and the enhanced power radiation is due to resonances in materials surrounding the antennas, whether a slot antenna or a dipole antenna, and irrespective of the coating material types.

The above studies, on material and metamaterial resonances, investigated the radiated power of the antennas, and did not study other antenna parameters. Thus, the problem of antennas with material or metamaterial coatings was not fully investigated then. This investigation was undertaken recently, by the author and his research group, by studying the problem of coated slot antennas in more detail, and determining their radiation patterns, as well as, the directivities [10–13]. For the antenna configuration, again a slotted spherical geometry was selected, so that its electromagnetic problem could be solved analytically, and without enforcing any approximation. Thus, the antenna parameters could be determined exactly, and anomalous behavior could not be blamed on approximations in the solutions. This meant, all resulting solutions must be satisfied by sound physical principles, and be explained satisfactorily. Also, to include the metamaterial effects, both material and metamaterial coatings were considered and investigated. It was shown that the antenna directivities depend on the resonating modes and did not change dramatically. The radiated power could also be explained by the conservation of the energy law. The results of these investigations were recently summarized in a paper by M. Ng Mou Kehn [14]. Note also that the slotted spherical antenna is, in effect, a spherical dipole antenna, and when coated, it becomes a spherical dipole antenna coated with materials. Thus, the performance of coated spherical antennas can be compared with those of its dipole counterpart.

To fully explain the resonance effects in antenna coatings and their influence on the antenna performance, both cases of material and metamaterial coatings are reviewed and studied together in this paper. The case of the material coating is investigated first, as it deals with a single layer problem and is easier to handle mathematically, and explain its solution and consequences. The case of paired material/metamaterial coating is investigated next, through the use of a dual layer coating to represent material/metamaterial pair, and its solution and consequences are compared with the former case of a single layer coating. It is shown that the results are identical in both cases, although the paired material/metamaterial coating can offer resonances with any coating thickness, provided suitable metamaterial is available. Of course, this is just an ideal metamaterial assumption, since practical metamaterials will have finite thicknesses and are highly lossy. For this reason, the effect of losses in the coating materials is also investigated in the single layer case, indicating significant dampening of the resonances, especially for higher order modes.

II. Case I: Slotted Spherical Dipole Antenna Coated With Homogeneous Materials

The geometry of the antenna is shown in Fig. 1. It is a spherical dipole antenna of radius a, coated with a homogeneous material of radius b, having the permittivity of ε and permeability of μ, placed in free space with parameters ε_o and μ_o, respectively. The sphere has a narrow slot at $\theta = \theta_o$, across which there is an applied voltage V, which is essentially an impulse function, defined by

$$E_\theta\big|_{r=a} = (V/a)\delta(\theta - \theta_o) \tag{1}$$

where δ is the Dirac delta function. With such an excitation the radiated field is transverse magnetic field to the radial direction, TM^r, and can be determined entirely from the magnetic vector potential A_r [3] and [5], which will have the E_θ and H_ϕ components only. Selecting appropriate vector potentials for the coating and free space regions, determining the electric and magnetic field components, and applying the boundary conditions on the sphere coating surfaces, the radiated field outside the coating can be shown to be of the form

$$E_\theta = \frac{1}{r}\sum_{n=1}^{\infty} C_n \hat{H}_n^{(2)\prime}(k_o r) P_n'(\cos\theta)\sin\theta \tag{2}$$

where

$$\hat{H}_n^{(2)}(k_o r) = (\pi k_o r/2)^{1/2} H_{n+1/2}(k_o r) \tag{3}$$

is the Riccati type spherical Hankel function of order n and $P_n(\cos\theta)$ is the nth order Legendre function. The prime on the Hankel and Legendre functions represents the derivative with respect to their arguments, and the constant C_n is given by,

$$C_n = -jV\left[(2n+1)/2n(n+1)\right]P_n'(\cos\theta_o)\sin\theta_o \left[\eta_r A_n \hat{H}_n^{(2)}(k_o b) - B_n \hat{H}_n^{(2)\prime}(k_o b)\right]^{-1} \tag{4}$$

with

$$A_n = \hat{J}_n'(ka)\hat{Y}_n'(kb) - \hat{J}_n'(kb)\hat{Y}_n'(ka) \tag{5}$$

and

$$B_n = \hat{J}_n'(ka)\hat{Y}_n(kb) - \hat{J}_n(kb)\hat{Y}_n'(ka) \tag{6}$$

where $\hat{J}_n$ and $\hat{Y}_n$ are the Riccati type spherical Bessel and Neumann functions of order n and k and η_r are the propagation constant and relative impedance of the coating material. Thus, the time average Poynting vector in the radial direction is given by

$$P = \tfrac{1}{2} Re(\boldsymbol{E} \times \boldsymbol{H}^*)\cdot U_r = \tfrac{1}{2}\left|E_\theta\right|^2/\eta_o \tag{7}$$

where U_r is the unit vector in the radial direction and η_o is the intrinsic impedance of free space. An integration of the Poynting vector over a sphere in the radiation zone gives the total radiated power, which can be shown to be

$$P = \frac{\pi V^2}{\eta_o}\sum_{n=1}^{\infty}\frac{2n+1}{2n(n+1)}\frac{\left|\sin\theta_o P_n'(\cos\theta_o)\right|^2}{\left|\Delta_n\right|^2} \tag{8}$$

with

$$\Delta_n = j\eta_r A_n \hat{H}_n^{(2)}(k_o b) - jB_n \hat{H}_n^{(2)\prime}(k_o b) \tag{9}$$

and the antenna Directivity becomes

$$D_o(\theta) = \frac{\left|\sum_{n=1}^{\infty} j^n C_n \sin\theta P_n'(\cos\theta)\right|^2}{\sum_{n=1}^{\infty}\frac{n(n+1)}{2n+1}\left|C_n\right|^2} \tag{10}$$

If only one mode, i.e. mode n, dominates the radiated power and the Directivity of the antenna reduces to

$$P = \frac{\pi V^2}{\eta_o}\frac{2n+1}{2n(n+1)}\frac{\left|\sin\theta_o P_n'(\cos\theta_o)\right|^2}{\left|\Delta_n\right|^2} \tag{11}$$

and

$$D_o(\theta) = \frac{2n+1}{n(n+1)}\left|\sin\theta P_n'(\cos\theta)\right|^2 \tag{12}$$

In the case of $b = a$, i.e. slotted spherical antenna with no coating, the radiated power from equation (8) reduces to

$$P_o = \frac{\pi V^2}{\eta_o} \sum_{n=1}^{\infty} \frac{2n+1}{2n(n+1)} \frac{\left|\sin\theta_o P_n'(\cos\theta_o)\right|^2}{\left|H_n^{(2)\prime}(k_o a)\right|^2} \tag{13}$$

which is the radiated power of the slotted spherical antenna without the coating. Thus, the ratio of P/P_o is an indication of the coating effect on the radiated power, which is investigated in the next section for different parameters of the coating material and the electrical size of the sphere. It should be noted again that the slotted spherical antenna of Fig. 1, has physically the same configuration as a spherical dipole antenna excited by a gap voltage, and P/P_o represents the effect of coating material on the radiated power of a dipole antenna.

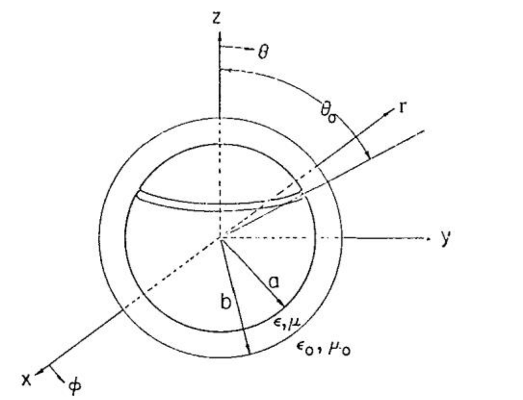


Fig. 1. Geometry of a spherical dipole antenna with a narrow excitation slot at $\theta = \theta_o$ and coated by a homogeneous material.

III. Numerical Results, Case I

Equation (8) shows the dependence of the radiated power on the slot and coating parameters. The slot effect is indicated by the angle θ_o, which in general represents an asymmetric dipole antenna. The case of θ_o= 90° makes the dipole excitation symmetrical and the antenna becomes equivalent to a practical dipole antenna. This parameter does not depend on the coating. The influence of the coating is represented by the function Δ_n, which is more complex and depends on the coating parameters. To understand its effect on the radiated power it is modified to,

$$\begin{aligned}\Delta_n &= \left[\eta_r A_n \hat{Y}_n(k_o b) - B_n \hat{Y}_n'(k_o b)\right] \\ &\quad + j\left[\eta_r A_n \hat{J}_n(k_o b) - B_n \hat{J}_n'(k_o b)\right]\end{aligned} \tag{14}$$

which for a lossless material separates the real and imaginary parts of Δ_n, and makes it easier to understand its variation. For example, in general, a complex function may not have a zero, since its real and imaginary parts may not become zero at the same time. This is an important property here and influences the effect of Δ_n on the radiated power. For a selected value of n, the real and imaginary parts increase or decrease with the coating radius b and have possible zeros along the real and imaginary axes. On the other hand, for $n < k_o b$, the magnitudes of the Riccati Bessel and Neumann functions are of the order of unity and any zeros of the real and imaginary parts of Δ_n have small effect on the radiated power. However, for $n > k_o b$ the magnitude of the Riccati Bessel functions decreases rapidly, decreasing significantly the imaginary part of Δ_n. In such cases, any zero of the imaginary part will not have significant effect of the radiated power. Instead, any possible zero of the real part will cause a partial resonance of the respective mode in the coating and a considerable increase of the radiated power. The radiation will then be primarily due to this mode, which also influences the shape of the radiation patterns. More interestingly, as shown in equation (12), the antenna Directivity becomes independent of its geometry and coating parameters.

To show the possibility of these resonances the radiated power of the coated antenna is computed and shown in Figs. 2 to 6. In all cases the vertical axis is the power ratio P/P_o, to compare the radiated powers of the coated and the uncoated cases, and the horizontal axis is the d/λ_c, where $d = b–a$ is the coating thickness and λ_c is the wavelength in the coating. The results for both dielectric and permeable type materials are presented and show numerous resonances, indicated by the mode number n on each resonance. Fig. 2 shows the results for a sphere of size $k_o a$ = 1.0 and θ_o = 90°. This size represents approximately the size of a half wave spherical dipole antenna. For this symmetric excitation of the antenna all even modes cancel out and the radiation is due to odd modes only, and the resonances are also due to the odd modes.

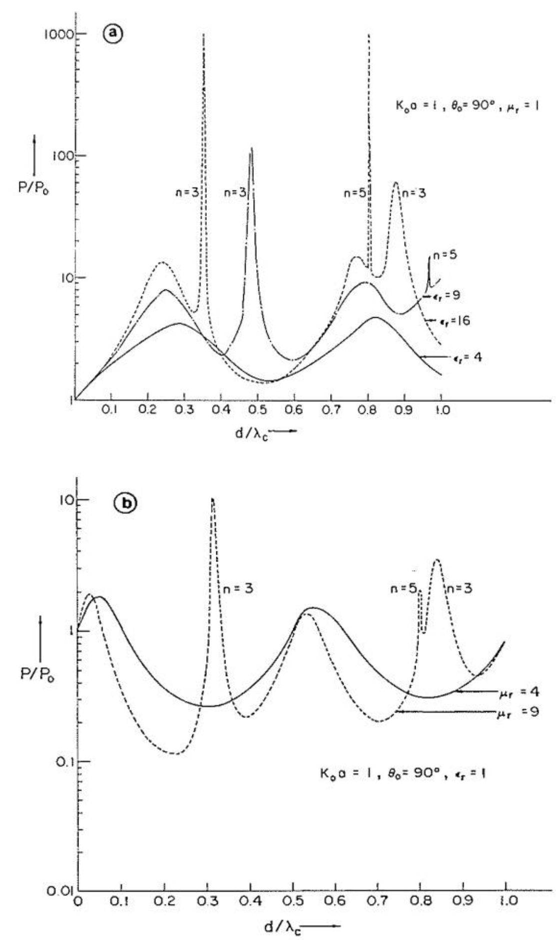


Fig. 2. Effect of the coating thickness d ($d = b–a$) on the radiated power of a coated spherical dipole antennas, $k_o a$ = 1.0 and θ_o = 90°, (a) μ_r = 1, (b) ε_r = 1.0.

In the dielectric coating case the radiated power, in Fig. 2(a), gradually increases with the coating thickness up until about d/λ_c = 0.25, then oscillates between resonances, and the resonances become more pronounced by increasing the relative dielectric constant. Similar results are also obtained for the permeable coating and shown in Fig. 2(b). However, since in this case the relative impedance η_r of the coating is larger than unity, the first term of the real and imaginary parts of Δ_n become much larger and the radiated power in between resonances reduces below that of the no coating case.

The corresponding results for a larger sphere with k_oa = 2.0 (Full wave dipole antenna) are shown in Figs. 3 and 4. Fig. 3 shows the results for θ_o = 60°, while Fig. 4 shows the same for θ_o = 90°. Since in the case of Fig. 3 the excitation is asymmetric, all modes make contribution to the radiated power and show their resonances, i.e. more resonances are detected for this case. However, the magnitude of the radiated powers are influenced by the Legendre functions, and for the θ_o = 90° case, are generally higher.

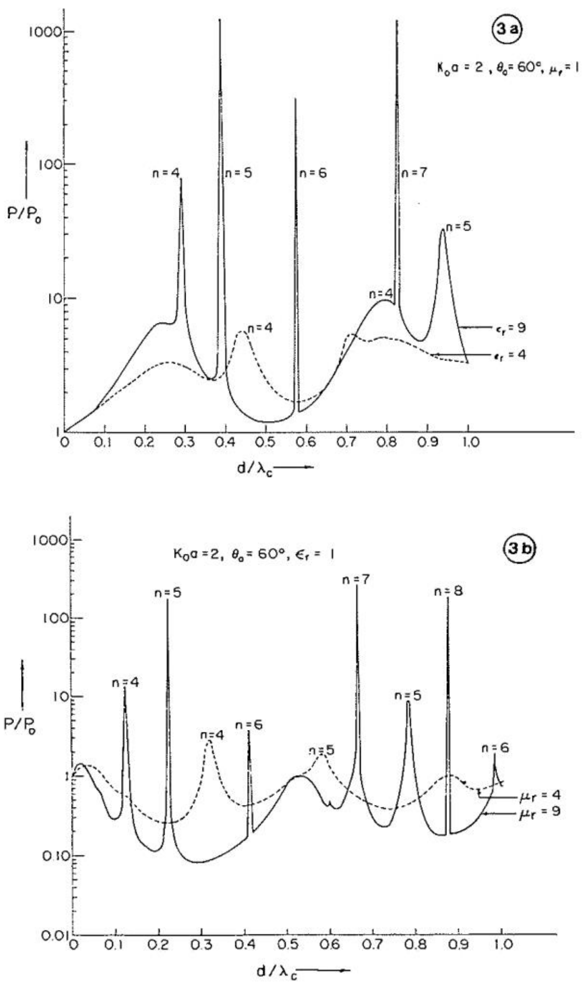


Fig. 3. Effect of the coating thickness d (d = b–a) on the radiated power of a coated spherical dipole antennas, k_oa = 2.0 and θ_O = 60°, (a) μ_r = 1, (b) ε_r = 1.0.

From the above results, as indicated earlier, the radiated power increases gradually, as the coating thickness increases from zero. In this range, the zeros of the real part of Δ_n occur for small values on n, for which the imaginary part of Δ_n is not significantly small, of course, depending the value of k_oa. As the coating thickness increases, the real part of Δ_n for higher order modes tend to become zero, while the imaginary part of Δ_n becomes very small, thus, resulting in sharp resonances at zeros of the real part. It was found that, in general, the first sharp resonance occurred at a thickness of about k_ob = k_oa + 2.

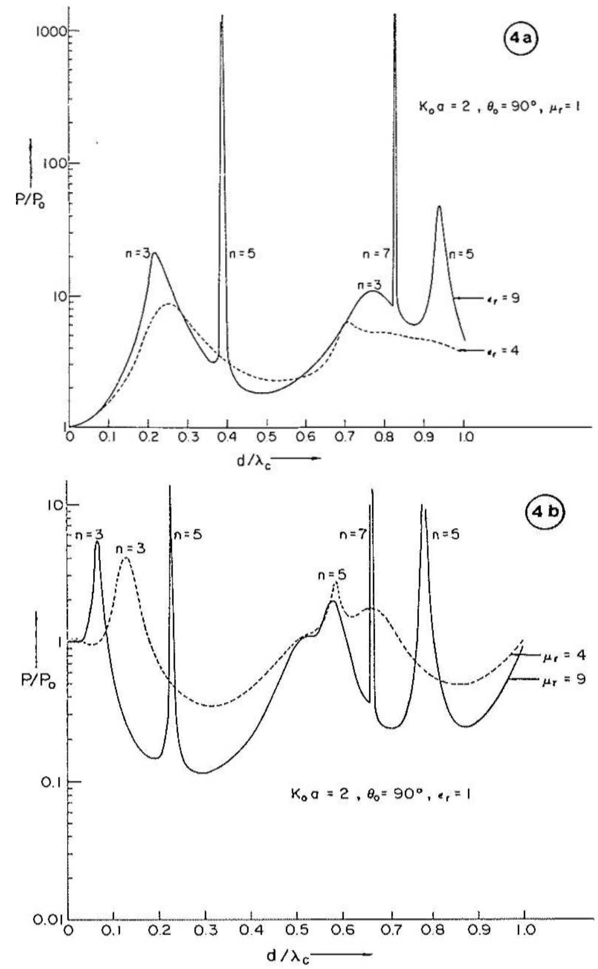


Fig. 4. Effect of the coating loss (tan δ = 0.25) and thickness d (d = b–a) on the radiated power of a coated spherical dipole antennas, k_oa = 2.0 and θ_O = 90°, (a) μ_r = 1, (b) ε_r = 1.0.

In all above cases the coating material was assumed to be lossless. In general, however, all natural materials are lossy. Thus, the material loss effect on the mode resonances was also investigated. For this case, a moderate loss level of tan δ = 0.02 was assumed, which is a reasonable assumption and common substrates, such as FR4, has loss tangent of the same order. The results are shown in Figs. 5 and 6 for both θ_o = 60° and 90°. As expected, the resonance effects are reduced significantly, especially for higher order modes. This is also expected, as the resonances are due to the modes propagating within the coating region, and their attenuation increases with mode order. Consequently, as shown in Figs. 5 and 6, the resonances beyond 5 are too small to be detected. However, the location of the resonances remain nearly the same, primarily, because the real parts of the relative permittivity and permeability of the coating material are much lager than the imaginary parts, and have remained the same as the lossless case.

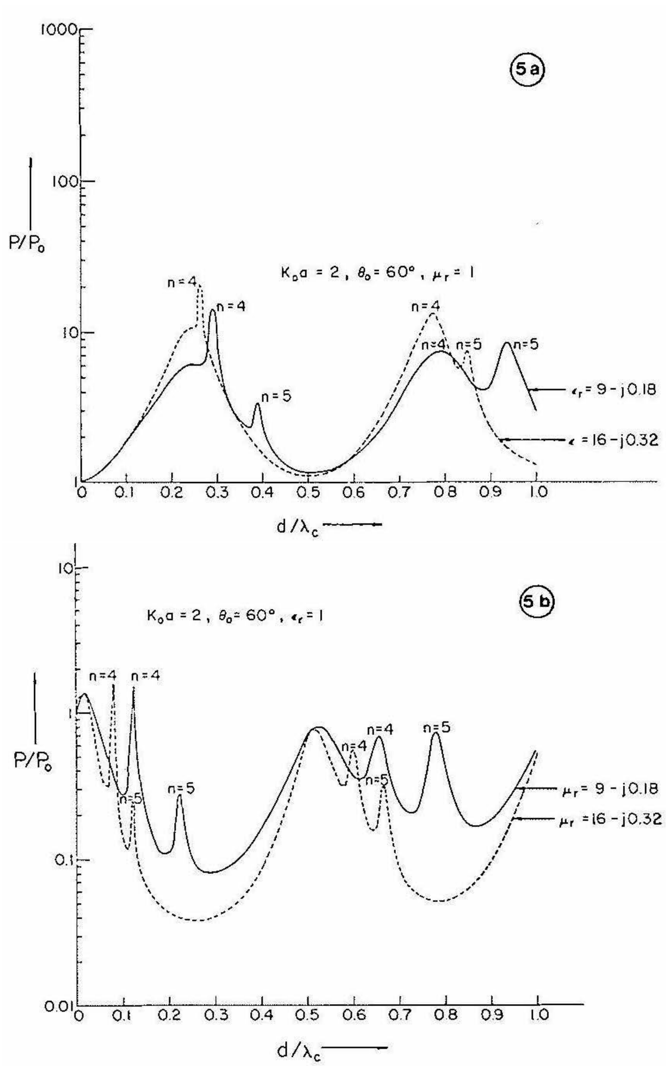


Fig. 5. Effect of the coating loss (tan $\delta = 0.25$) and thickness d ($d = b–a$) on the radiated power of a coated spherical dipole antennas, $k_oa = 2.0$ and $\theta_o = 60°$, (a) $\mu_r = 1$, (b) $\varepsilon_r = 1.0$.

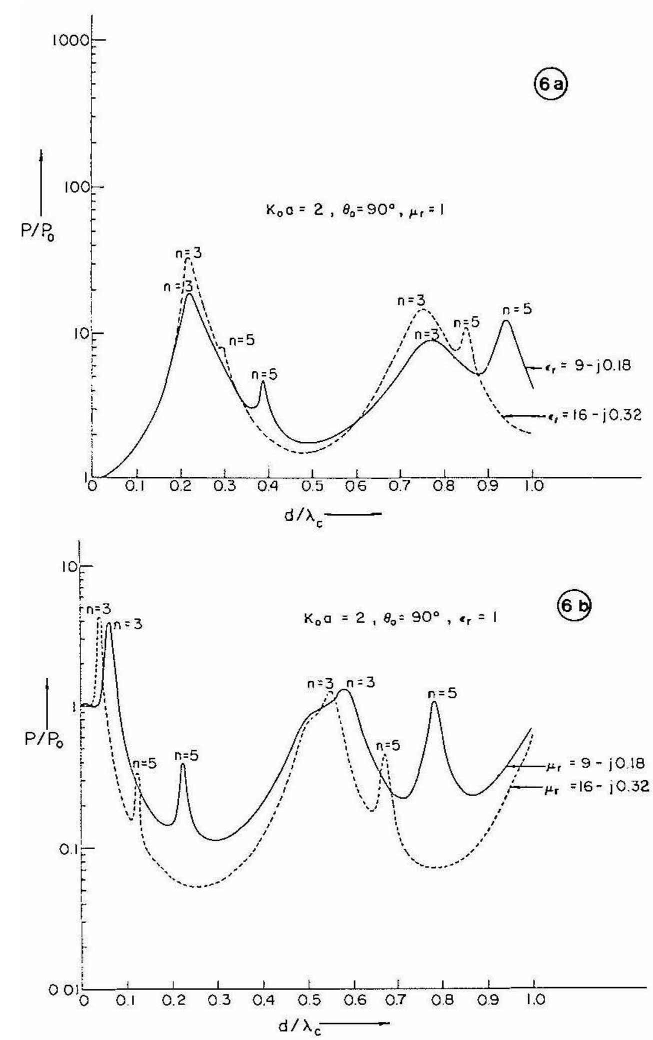


Fig. 6. Effect of the coating loss (tan $\delta = 0.25$) and thickness d ($d = b–a$) on the radiated power of a coated spherical dipole antennas, $k_oa = 2.0$ and $\theta_o = 90°$, (a) $\mu_r = 1$, (b) $\varepsilon_r = 1.0$.

IV. Discussion of the Results, Case I

The geometry of the problem is a spherical dipole antenna, excited by a gap voltage V, which is represented by a slotted conducting sphere with the voltage V applied across its slot. This is a very practical antenna problem and $k_oa = 1.0$ is a practical size, approximating a half wave dipole. The results in Figs. 2 to 4 show radiated power enhancement, with coating, for dielectric type materials, and impulse type radiated power increase for both permittivity and permeability type materials. These results were given above, but no physical explanations were provided. Also, the sharp peaks were indicated to be due to mode resonances but no proof were given. Both these questions are addressed here. First we consider the mode resonances.

In the coating region the electromagnetic field satisfies the following boundary conditions.

$$E_q^c = 0 \text{ for } r = a \tag{15}$$

and

$$H_\phi^a = \frac{1}{r}\sum_{n=1}^{\infty} c_n \hat{H}_n^{(2)}(k_o r)\sin\theta P_n'(\cos\theta) \qquad (16)$$

where the superscripts *c* and *a* refer to the coating and free space, respectively. Now, since the excitation is due to a constant voltage across the slot, which is independent of the azimuth angle ϕ, the resulting fields have the components E_θ and H_ϕ. The solutions for E_θ have the form provided in equation (2). Similarly, the H_ϕ can be shown to have solutions of the forms

$$H_\phi^c = \frac{1}{r}\sum_{n=1}^{\infty}\left[a_n \hat{H}_n^{(1)}(kr) + b_n \hat{H}_n^{(2)}(kr)\right]\sin\theta P_n'(\cos\theta) \qquad (17)$$

Enforcing the boundary conditions for continuity of tangential components at $r = a$ and $r = b$ provides the following equations.

$$\eta_r\left[a_n \hat{H}_n^{(1)\prime}(ka) + b_n \hat{H}_n^{(2)\prime}(ka)\right] = 0 \qquad (18)$$

$$\eta_r\left[a_n \hat{H}_n^{(1)\prime}(kb) + b_n \hat{H}_n^{(2)\prime}(kb)\right] = c_n \hat{H}_n^{(2)\prime}(k_o b) \qquad (19)$$

$$\left[a_n \hat{H}_n^{(1)}(kb) + b_n \hat{H}_n^{(2)}(kb)\right] = c_n \hat{H}_n^{(2)}(k_o b) \qquad (20)$$

where a_n, b_n and c_n are the solution constant to be determined. The solution of these equations gives the resonant modes in the coating region. However, equations (18) to (20) have a nontrivial solution only if the determinant of the coefficients becomes zero, which gives

$$j\eta_r A_n \hat{H}_n^{(2)}(k_o b) - jB_n \hat{H}_n^{(2)\prime}(k_o b) = 0 \qquad (21)$$

where A_n and B_n are given by equations (5) and (6). This equation, i.e. equation (21), is the same as Δ_n, provided in equation (9). Thus, the sharp peaks in the radiated power are due to the resonant modes in the coating region. Note that, equation (21) is obtained for the source free fields in the coating, i.e. $V = 0$. This is expected, since by definition, the resonant modes are the solution of source free field equations. Also, in solving the equations (18) to (20), two of the coefficients are determined in terms of the third, which becomes an arbitrary constant. For the antenna problem of Fig. 1, this arbitrary constant was determined in terms of the applied voltage *V*. On the other hand, the boundary condition (14) requires that the voltage across the slot should also vanish, i.e. $V = 0$, meaning the excitation voltage to become zero. However, since in the original problem of Fig. 1, the excitation voltage was finite, the radiated powers at the onset of the resonant modes become infinite. From the circuit point of view, the excitation source becomes equivalent to a current source with infinite source impedance. However, practical sources have finite impedances, normally 50 ohm, and the excitation voltage changes with the coating thickness, i.e. in practical problems the radiated power never becomes infinite.

The excitation uncertainty at mode resonances can be overcome by considering the fact that the coefficients of the resonant modes become significantly larger than those of the other modes, and the radiated field can be represented only by the resonant mode alone. This means, in equation (2) the summation drops out and only the mode *n* remains. Thus, the radiated power P_{rad} will be given by equation (11) for only a single mode *n*, and for a lossless coating the power relationship becomes

$$P_{rad} = P_{in} = \left(P = \frac{\pi V^2}{\eta_o}\,\frac{2n+1}{2n(n+1)}\,\frac{\left|\sin\theta_o P_n'(\cos\theta_o)\right|^2}{\left|\Delta_n\right|^2}\right) \qquad (22)$$

where P_{in} is the source power delivered to the antenna, i.e. it is the source power minus the reflected power. Equation (22) provides the correct value of the excitation voltage *V* in terms of the input power P_{in} of the antenna, and the energy conservation law is preserved. Note that in such a case, the resonant mode uniquely defines the radiation pattern of the antenna also. In particular, the Directivity of the antenna simplifies to the Directivity provided by the mode *n*, as given by equation (12), repeated here to facilitate discussion.

$$D_o(\theta) = \frac{2n+1}{n(n+1)}\left|\sin\theta P_n'(\cos\theta)\right|^2 \qquad (23)$$

It is interesting to note that, under the mode resonance in the coating region, the antenna shape and size do not affect its radiation patterns, radiated power, and the Directivity. The resonant mode completely defines the antenna radiation performance. Table 1 shows the antenna Directivity at $\theta = 90°$, i.e. the horizontal plane, for the first seven modes. Since the excitation is for $\theta_o = 90°$, all even modes cancel out and the radiation is due to the odd modes only. It is interesting to note that, the directivity in the horizontal plane decreases with the mode number *n*. It is maximum for $n = 1$, i.e. the first resonant mode, where it is $D_o = 1.5$ (1.76 dBi). Thereafter, it decreases gradually to 1.276 (1.058 dBi) for mode $n = 11$. Thus, the coating actually decreases the antenna Directivity in the horizontal plane, even though it increases the size of the antenna, and the larger the antenna the smaller the Directivity becomes. However, as shown in Fig. 7, the peak of the radiation pattern moves gradually away from the horizontal axis, and towards the *z*-axis.

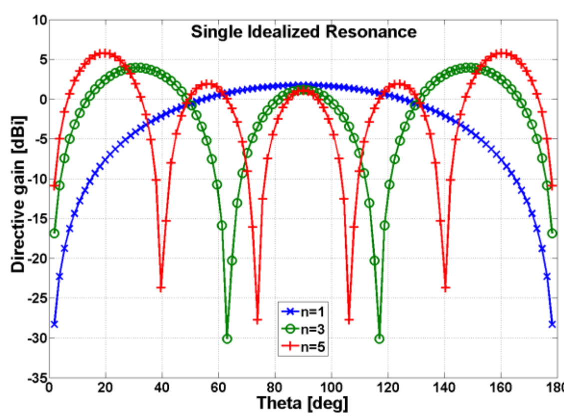


Fig. 7. Radiation patterns of resonant modes for $n = 1$, 3, and 5, $\theta_o = 90°$.

V. Case II : Slotted Spherical Dipole Antenna Coated With Material and Metamaterial

The geometry of this antenna is shown in Fig. 8, which is the same spherical dipole antenna of Fig. 1, but now coated with two layers of materials. The inner layer is designated as

Layer 1 and has the permittivity ε_1 and permeability μ_1, while the outer layer is Layer 2 with parameters ε_2 and μ_2. These parameters can in general be positive, negative, real or imaginary, to represent real homogeneous material coating, or homogeneous metamaterial coating. This case is the same as that in Case I, except now there is two layers of coating materials. Thus, the problem can be solved in a similar fashion, by selecting appropriate vector potentials as

$$A_r^{(1)} = \frac{V}{a}\mu_o\varepsilon_o\sum_{n=1}^{\infty}\left[a_n^{(1)}\hat{J}_n(k_1 r) + b_n^{(1)}\hat{Y}_n(k_1 r)\right] P_n(\cos\theta) \quad (24a)$$

$$A_r^{(2)} = \frac{V}{a}\mu_o\varepsilon_o\sum_{n=1}^{\infty}\left[a_n^{(2)}\hat{J}_n(k_2 r) + b_n^{(2)}\hat{Y}_n(k_2 r)\right] P_n(\cos\theta) \quad (24b)$$

$$A_r^{(0)} = \frac{V}{a}\mu_o\varepsilon_o\sum_{n=1}^{\infty} c_n \hat{H}_n^{(2)}(k_0 r) P_n(\cos\theta) \quad (24c)$$

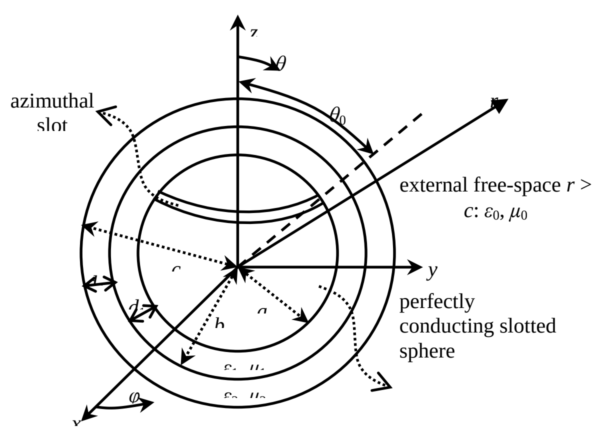


Fig. 8. Geometry of a spherical dipole antenna with a narrow azimuthal excitation slot and coated with two layers of homogeneous materials.

where the superscripts 1, 2 and 0 refer to each coating layer and free space, and the constants a_n, b_n and c_n are the mode coefficients in each region, similar to Case I. The boundary conditions too are the continuity of the tangential components of the electric and magnetic fields between the layers, and vanishing of the tangential electric field on the sphere, as before. Enforcing these boundary conditions the mode coefficients can be determined as in the Case I, except that the determinant of the coefficient matrix now is given by,

$$\Delta_n = \begin{vmatrix} \hat{J}_n'(k_1 a) & \hat{Y}_n'(k_1 a) & 0 & 0 & 0 \\ \hat{J}_n'(k_1 b) & \hat{Y}_n'(k_1 b) & -\sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\hat{J}_n'(k_2 b) & -\sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\hat{Y}_n'(k_2 b) & 0 \\ 0 & 0 & \hat{J}_n'(k_2 c) & \hat{Y}_n'(k_2 c) & -\sqrt{\frac{\mu_2\varepsilon_2}{\mu_o\varepsilon_o}}\hat{H}_n^{(2)\prime}(k_o c) \\ \mu_2\hat{J}_n(k_1 b) & \mu_2\hat{Y}_n(k_1 b) & -\mu_1\hat{J}_n(k_2 b) & -\mu_1\hat{Y}_n(k_2 b) & 0 \\ 0 & 0 & \mu_o\hat{J}_n(k_2 c) & \mu_o\hat{Y}_n(k_2 c) & -\mu_2\hat{H}_n^{(2)}(k_o c) \end{vmatrix} \quad (25)$$

Again, the radiated power can be shown to be

$$P_{rad} = \frac{\pi V^2}{\eta_o}\sum_{n=1}^{\infty}|c_n|^2\frac{n(n+1)}{2n+1} \quad (26)$$

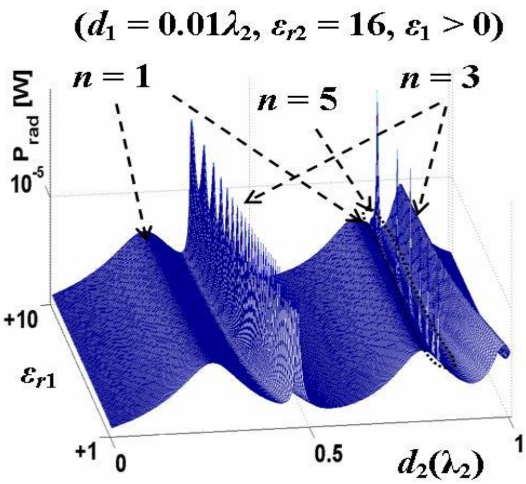
and the antenna Directivity will be given by

$$D_o(\theta) = \frac{\left|\sum_{n=1}^{\infty}\sin\theta P_n'(\cos\theta)e^{jn\pi/2}\right|^2}{\sum_{n=1}^{\infty}|c_n|^2\frac{n(n+1)}{2n+1}} \quad (27)$$

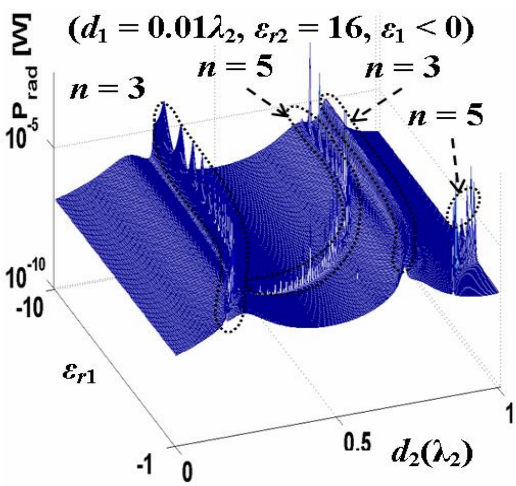
Since these equations are identical to those in Case I, at mode resonances the radiated power and the antenna Directivity will be given by the above expressions, without the summation signs, which were shown in Case I, in equations (11) and (12), and are not repeated here.

VI. Numerical Results, Case II

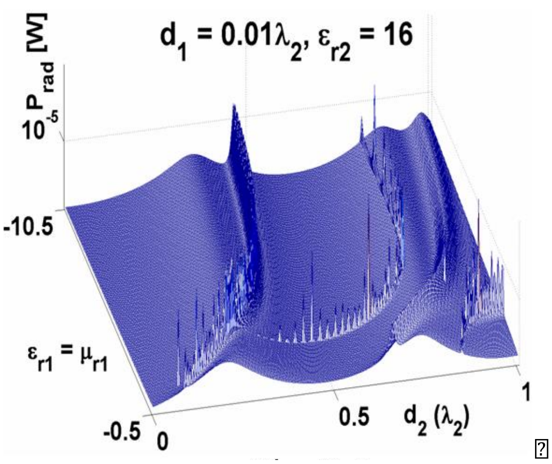
Although the solution in this case follows the same trend as those in Case I, a few differences exist that are shown below and discussed. Figs. 9(a) to 9(c) show the radiated power as a function of the coating parameters. In Fig. 9(a), the permittivity and permeability of both layers are positive, representing natural materials. Similar to the results of Fig. 2, at certain thicknesses of the coating the radiated power rises rapidly at the mode resonances. However, in this case the coating thickness must be significant to permit the mode resonance. The situation is different in Figs. 9(b) (with negative permittivity) and 9(c) (with double negative parameters). Mode resonances occur even for very small thicknesses of the coating. To investigate this phenomenon further the contour locations of the resonances are shown in Figs. 10(a) to 10(c). In these figures the vertical axis is the permittivity of Layer 1, which varies from the negative to positive values. They clearly show that for positive permittivity values the resonances occur only for certain large coating thicknesses. However, for negative permittivity, or double negative parameters, the resonances can occur even for very small thickness values.



(a)

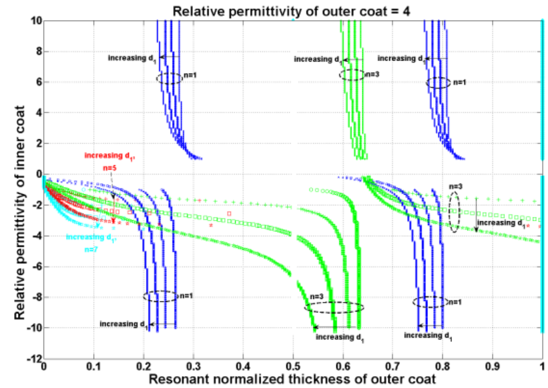


(b)

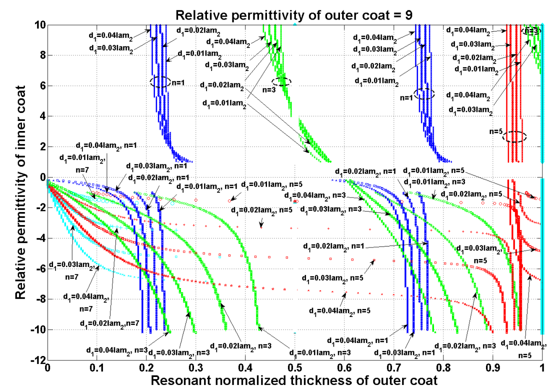


(c)

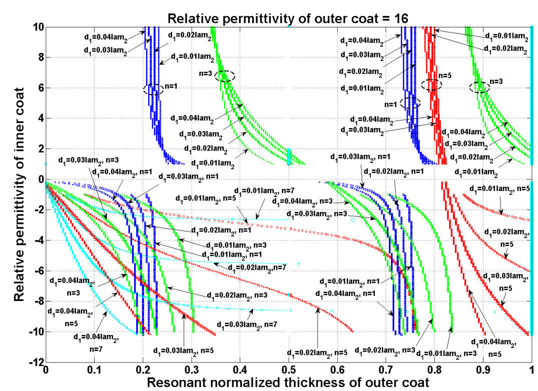
Fig. 9. Radiated power of spherical dipole antenna coated with two layers of homogeneous material, (a) all parameters are positive, (b) Layer 1 with negative permittivity, (c) Layer 1 with double negative parameters, θ_o= 90°.



(a)



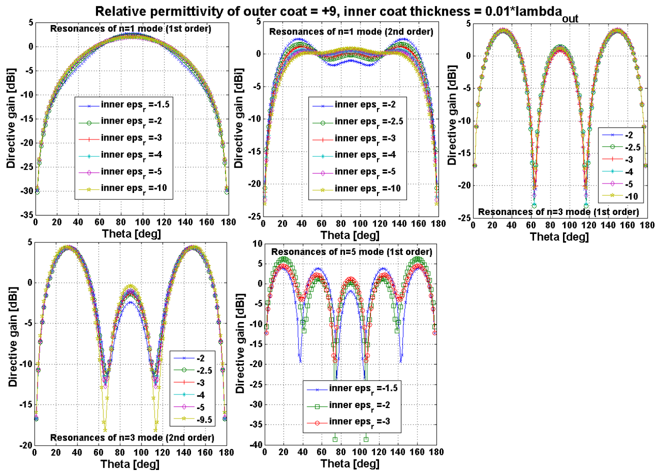
(b)



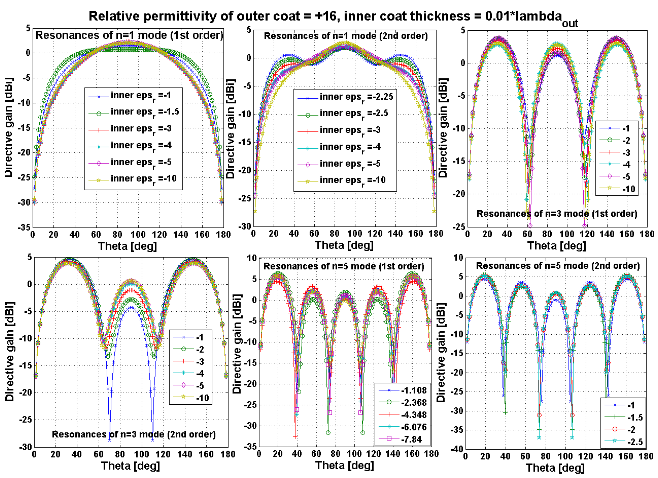
(c)

Fig. 10. Contour paths of the resonating modes for a double layer coating of Fig. 8, where the permittivity of Layer 1 changes from negative to positive values, θ_o = 90°.

Fig. 11 shows the radiation patterns for different coating parameters, when one of the modes, 1, 3 or 5, is resonant. It is clear that for widely different coating parameters, the radiation patterns remain nearly the same, as long as the same mode in resonant. Small variations in the radiation patterns are due to partial excitations of the adjacent modes. This is shown further in Figs. 12(a) to 12(c). In the case of modes 1 and 3, the mode purity is excellent. The excitation of the other modes is negligible. For the selected coating parameters to resonate mode 5, adjacent modes are also excited to some degree and affect the radiation patterns, as shown in Fig. 11(c).

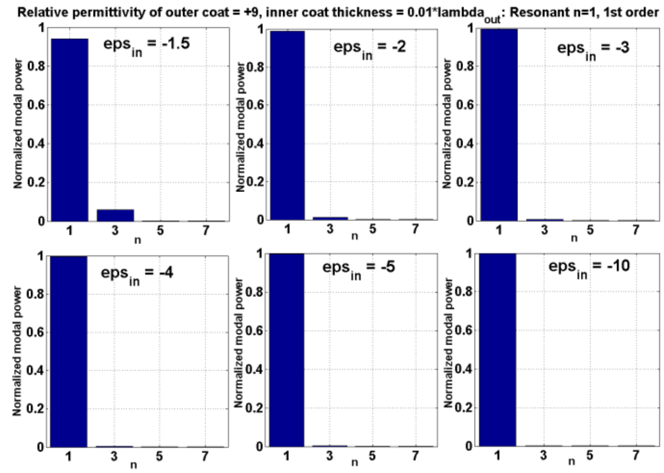


(a)

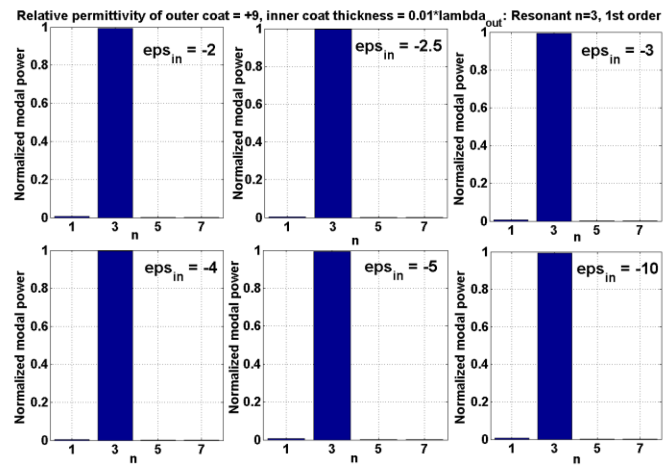


(b)

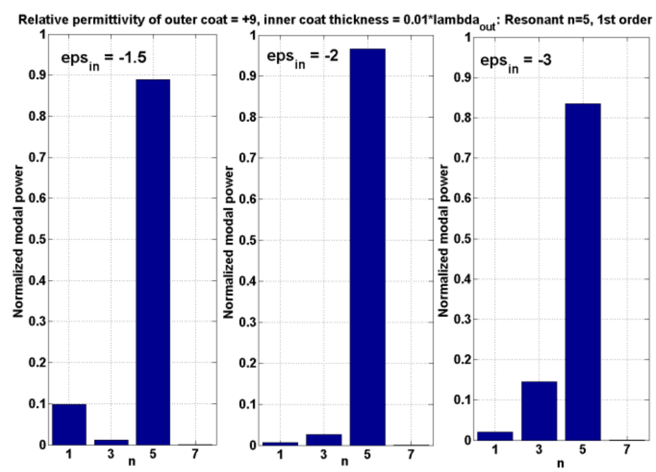
Fig. 11. Radiation patterns of the coated spherical dipole antenna with different coating parameters, but only a single mode resonating in the coating, (a) ε_2 = 9, (b) ε_2 = 16, θ_o = 90°.



(a)



(b)



(c)

Fig. 12. Relative radiated power levels of resonating and their adjacent modes of the spherical dipole antenna of Fig. 8, θ_o = 90°, (a) Fig. 11(a) and n = 1, (b) Fig. 11(a) and n = 3, (c) Fig. 11(a) and n = 5.

VII. Conclusions

The problem of a spherical dipole antenna, coated with a single or double homogeneous materials, and excited by a constant gap voltage was investigated. Due to the spherical geometry of the antenna, its electromagnetic problem could be solved analytically. The radiated power was investigated as a function of the coating thickness and electrical parameters. It was shown that for dielectric coatings, the radiated power increases by the thickness of the coating, and becomes infinite for certain thicknesses. By solving for the guided modes inside the coating region, it was shown that these thicknesses correspond to mode resonances inside the coating material. The problem was also investigated for permeable materials, for which the radiated power decreased with the coating thickness, except at the mode resonances. Selecting the resonating mode to represent the field solution, the antenna radiated power and Directivity were expressed in terms of the source power. This eliminated the ambiguity about the radiated power and antenna Directivity. In particular, it was shown that the resonating modes completely determine the antenna radiation parameters, and the antenna or coating size and parameters do not influence the results for radiated power and Directivity.

The problem was also investigated for the antenna coated with metamaterials. A dual layer coating was selected and by solving the boundary value problem the radiated power and antenna Directivity, as well as, the mode resonances were determined. It was shown, again, that the sharp increases in the radiated power were due to mode resonances inside the coating, and the radiated power and the antenna Directivity were governed by the same expressions as those for the single layer coating case, i.e. the enhancement of the radiated power by the coating was not due to the metamaterial. The main difference was, however, that with a pair of material and metamaterial, mode resonances could be obtained with even small thicknesses of the coating. This is, however, an ideal situation, as in practice for simulating metamaterial effects finite material thicknesses are necessary. To confirm that at mode resonances the resonating modes define the antenna radiation properties, for sample coating parameters, the mode coefficients were also computed. It was shown that in all cases the contributions of the other, or adjacent modes, to the radiated field were, generally very small, or negligible.

Table I - Directivity of the resonant mode n at $\theta_o = 90°$.

N	1	3	5	7	9	11
D_o	1.5	1.31	1.29	1.28	1.278	1.276
D_o(dBi)	1.76	1.18	1.10	1.07	1.065	1.058

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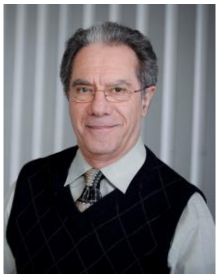

Lotfollah Shafai B.Sc. from University of Tehran in 1963 and M.Sc. and Ph.D., from University of Toronto, in 1966 and 1969. In November 1969, he joined the Department of Electrical and Computer Engineering, University of Manitoba as a Lecturer, Assistant Professor 1970, Associate Professor 1973, Professor 1979, and Distinguished professor 2002. His assistance to industry was instrumental in establishing an Industrial Research Chair in Applied Electromagnetics at the University of Manitoba in 1989, which he held until July 1994.

In 1986, he established the symposium on Antenna Technology and Applied Electromagnetics, ANTEM, at the University of Manitoba, which has grown to be the premier Canadian conference in Antenna technology and related topics.

He has been the recipient of numerous awards. In 1978, his contribution to the design of the first miniaturized satellite terminal for the Hermes satellite was selected as the Meritorious Industrial Design. In 1984, he received the Professional Engineers Merit Award and in 1985, "The Thinker" Award from Canadian Patents and Development Corporation. From the University of Manitoba, he received the "Research Awards" in 1983, 1987, and 1989, the Outreach Award in 1987 and the Sigma Xi Senior Scientist Award in 1989. In 1990 he received the Maxwell Premium Award from IEE (London) and in 1993 and 1994 the Distinguished Achievement Awards from Corporate Higher Education Forum. In 1998 he received the Winnipeg RH Institute Foundation Medal for Excellence in Research. In 1999 and 2000 he received the University of Manitoba Research Award. He is a life Fellow of IEEE and a life Fellow of The Royal Society of Canada. He was a recipient of the IEEE Third Millenium Medal in 2000 and in 2002 was elected a Fellow of The Canadian Academy of Engineering and Distinguished Professor at The University of Manitoba. In 2003 he received an IEEE Canada “Reginald A. Fessenden Medal” for “Outstanding Contributions to Telecommunications and Satellite Communications”, and a Natural Sciences and Engineering Research Council (NSERC) Synergy Award for “Development of Advanced Satellite and Wireless Antennas”. He holds a Canada Research Chair in Applied Electromagnetics and was the International Chair of Commission B of the International Union of Radio Science (URSI) for 2005-2008. In 2009 he was elected a Fellow of the Engineering Institute of Canada, and was the recipient of IEEE Chen-To-Tai Distinguished Educator Award. In 2011 he received the Killam Prize in Engineering from The Canada Council, for his “outstanding Canadian career achievements in engineering, and his research on antennas”. In 2013 he received The “John Kraus antenna Award” from IEEE Antennas and Propagation Society “For contributions to the design and understanding of small high efficiency feeds and terminals, wideband planar antennas, low loss conductors, and virtual array antennas”. In 2014 he was the recipient of Edward E. Altshuler Best paper Prize from IEEE APS Magazine.